Quadratic Equations and Graphs

1. Part of the curve with equation $y = x^2 - 5x + 3$ is drawn on the grid.

The equation of another curve is $y = -\frac{x^3}{6} + \frac{6x^2}{5} - \frac{3x}{2}$

(a) Complete the table of values for $y = -\frac{x^3}{6} + \frac{6x^2}{5} - \frac{3x}{2}$

Give your values of y to 2 decimal places.

x	0	0.5	1	2	3	4	4.5	5
y	0	–0.47	-0.47	0.47	1.8	2.53	2.36	1.67

(b) On the grid opposite, plot the points from your completed table and join them to form a smooth curve.

(c) Use the two curves on the grid to find an estimate, to 2 decimal places, of the range of positive values of x for which $\frac{x^3}{6} - \frac{x^2}{5} - \frac{7x}{2} + 3 < 0$

Show your working clearly. $0.85 < x < 4.85$

For positive values of x, the two curves on the grid intersect at the points P and Q.

(d) Find an estimate, to 1 decimal place, of the gradient of the straight line through P and Q.

The equation of the straight line through P and Q has the form $y = ax + b$

(e) Find, to 1 decimal place, the value of b.

d) At $(0.85, -0.5)$ & $(4.8, 1.9)$

$$m = \frac{1.9 + 0.5}{4.8 - 0.85} = 0.61$$

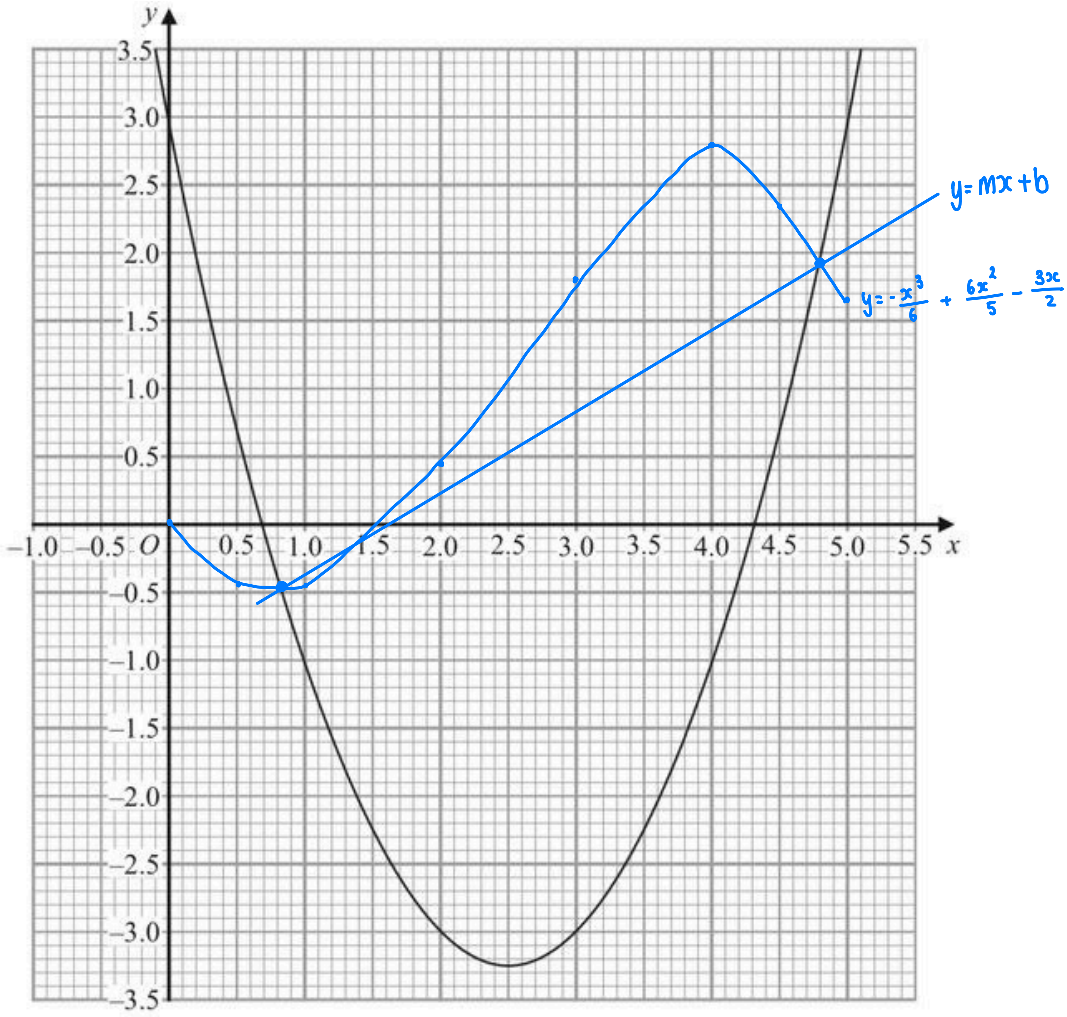
e) $y = mx + c$

At $(0.85, -0.5)$

$-0.5 = 0.61(0.85) + c$

$c = -1.0$

$\therefore y = 0.61x - 1.0$


y
x
O
3.5
3.0
2.5
2.0
1.5
1.0
0.5
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−1.0
−0.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
$y = mx + b$
$y = -\frac{x^3}{6} + \frac{6x^2}{5} - \frac{3x}{2}$

2. The equation of a curve C is $y = \dfrac{(2x-3)(kx+5)}{x}$, where k is a constant.

The point A on C is a stationary point.

Given that the x coordinate of A is $\dfrac{1}{2}$

find the value of k.

$$y = \frac{(2x-3)(kx+5)}{x}$$

$$= \frac{2kx^{\cancel{2}} + 10\cancel{x} - 3k\cancel{x} - 15}{\cancel{x}}$$

$$= 2kx + 10 - 3k - \frac{15}{x}$$

$$\frac{dy}{dx} = 2k + \frac{15}{x^2}$$

$$2k + \frac{15}{\left(\frac{1}{2}\right)^2} = 0$$

$$2k + \frac{15}{\frac{1}{4}} = 0$$

$$2k + 60 = 0$$

$$2k = -60$$

$$k = -30$$

3. Given that $-3x^2 + 6x + 2$ can be written in the form $a(x + b)^2 + c$ where a, b and c are integers, find the value of a, the value of b and the value of c.

$$-3x^2 + 6x + 2 = a(x+b)^2 + c$$

$$-3x^2 + 6x + 2 = ax^2 + 2abx + ab^2 + c$$

$$a = -3$$

$$2abx = 6x$$

$$2ab = 6$$

$$2(-3)b = 6$$

$$b = -1$$

$$ab^2 + c = 2$$

$$-3(-1)^2 + c = 2$$

$$-3 + c = 2$$

$$c = 5$$

$$\therefore a = -3,\ b = -1,\ c = 5$$

4. The curve C has equation

$$y = x^2 + 2x + \frac{4}{x} \qquad x \neq 0$$

(a) Complete the table of values for C

x	-4	-2	-1	-0.5	0.5	1	2	4
y	7	-2	-5	-8.75	9.25	7	10	25

(b) On the grid opposite, plot the points from your completed table.

The curve has one turning point and this has coordinates (1, 7)

(c) Use your points to draw the graph of $y = x^2 + 2x + \frac{4}{x} \qquad x \neq 0$

(d) Using your graph, find an estimate, to one decimal place, for the solution of the equation

$x = -2.6$

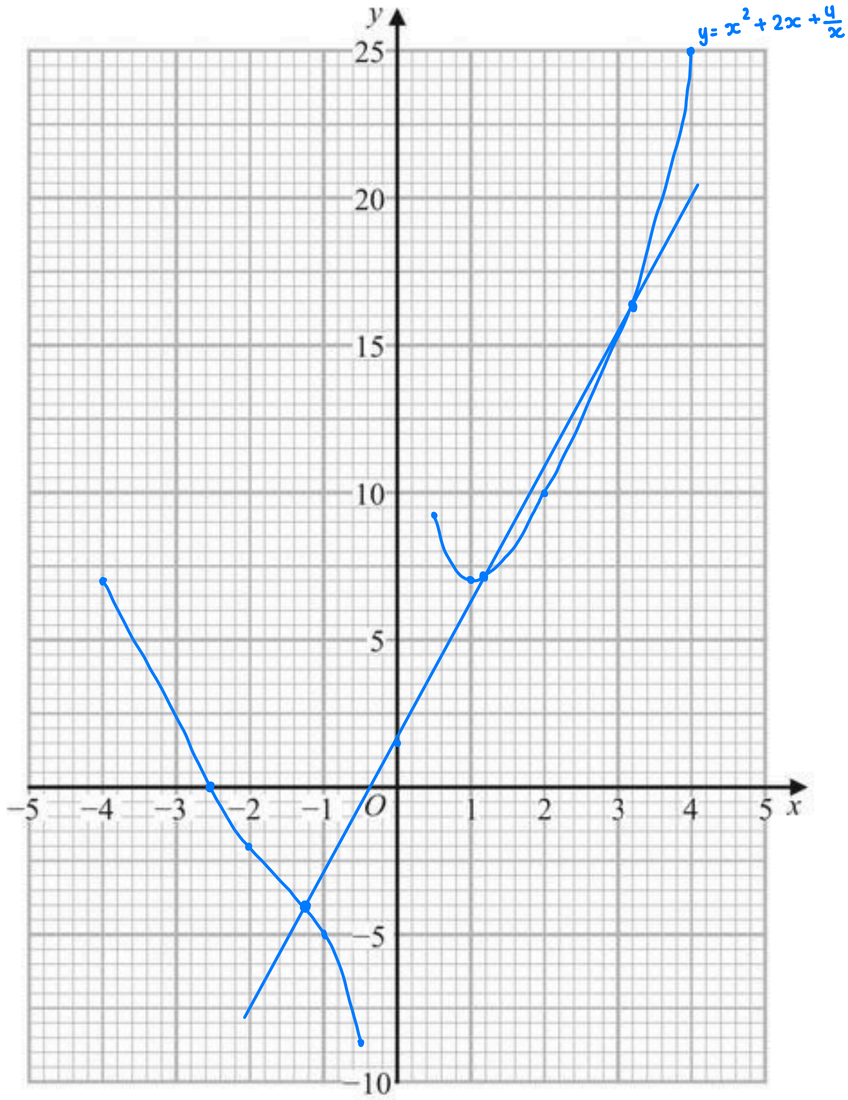
$$x^2 + 2x + \frac{4}{x} = 0$$

(e) By drawing a suitable line on the grid estimate, to one decimal place, the solutions of the equation

(0, 1.5)

$$x^2 + 2x + \frac{4}{x} = 5x + \frac{3}{2}$$

$x = -1.2, 1.1, 3.1$



y
25
20
15
10
5
−5
−10
−5
−4
−3
−2
−1
O
1
2
3
4
5
x
$y = x^2 + 2x + \frac{4}{x}$

5. The equation of a curve C is $y = x^2 - \frac{3}{2}x - 1$

The curve C has a minimum at the point A

(a) Show that the coordinates of A are $(0.75, -1.5625)$

(b) Complete the table of values for $y = x^2 - \frac{3}{2}x - 1$

x	−3	−2	−1	0	1	2	3	4
y	12.5	6	1.5	−1	−1.5	0	3.5	9

The point A has been plotted on the grid opposite.

(c) On the grid opposite, draw the curve with equation $y = x^2 - \frac{3}{2}x - 1$ for values of x from −3 to 4

(d) Using your curve, find an estimate, to one decimal place, for the range of values of x for which $x^2 - \frac{3}{2}x - 1 \leqslant 3$

Show your working clearly. $-1.4 \leq x \leq 2.9$

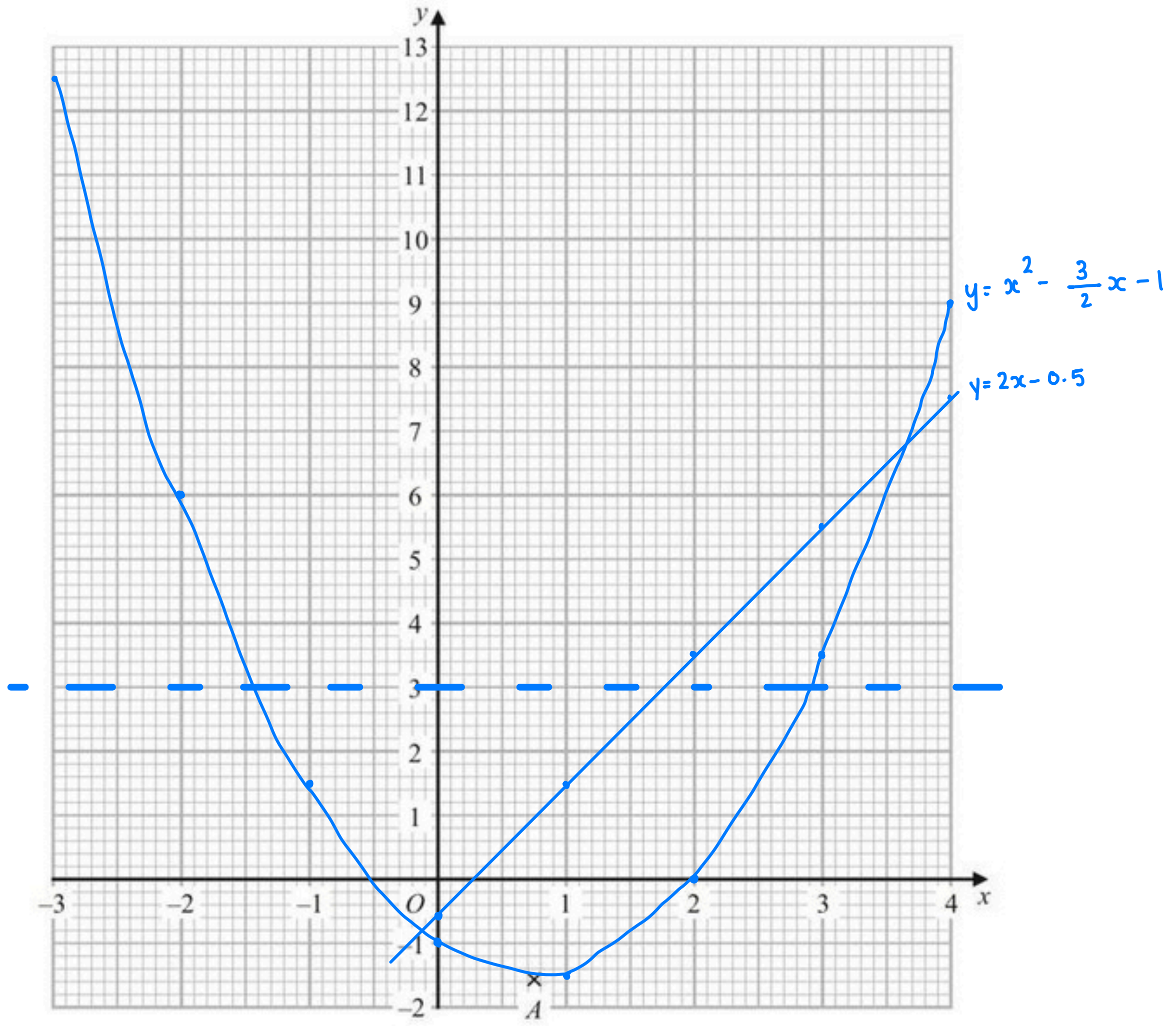
(e) By drawing a suitable straight line on the grid, find estimates, to one decimal place, of the solutions of the equation $x^2 - \frac{7}{2}x = \frac{1}{2}$

Show your working clearly.

$x^2 - \frac{3}{2}x - 1 = 2x - 0.5$

$y = 2x - 0.5$

$x = -0.1 \ , \ 3.6$


$y = x^2 - \frac{3}{2}x - 1$
$y = 2x - 0.5$
A

6. Given that, for all values of x,

$$8x^2 - 48x + 10 = a(x+b)^2 + c \text{ where } a, b \text{ and } c \text{ are integers,}$$

find the value of a, the value of b and the value of c.
Show your working clearly.

$$8x^2 - 48x + 10 = a(x+b)^2 + c$$

$$8x^2 - 48x + 10 = ax^2 + 2abx + ab^2 + c$$

$$a = 8$$

$$2ab = -48$$

$$2(8)b = -48$$

$$b = -3$$

$$ab^2 + c = 10$$

$$(8)(-3)^2 + c = 10$$

$$72 + c = 10$$

$$c = -62$$

$$a = 8,\ b = -3,\ c = -62$$

7. By writing $8x^2 - 56x + 17$ in the form $p(x+q)^2 + r$ find the exact solutions of the equation

$$8x^2 - 56x + 17 = 0$$

Give your answer in the form $\dfrac{a \pm b\sqrt{2}}{c}$ where a, b and c are integers.

Show your working clearly.

$$8x^2 - 56x + 17 = p(x+q)^2 + r$$

$$8x^2 - 56x + 17 = px^2 + 2qpx + pq^2 + r$$

$$p = 8$$

$$2qp = -56$$

$$2(8)q = -56$$

$$q = -3.5$$

$$pq^2 + r = 17$$

$$8(-3.5)^2 + r = 17$$

$$r = 17 - 98$$

$$= -81$$

$$8(x - 3.5)^2 - 81 = 0$$

$$(x - 3.5)^2 = \frac{81}{8}$$

$$x - 3.5 = \pm\sqrt{\frac{81}{8}}$$

$$x = \pm\sqrt{\frac{81}{8}} + 3.5$$

$$= \frac{9\sqrt{2}}{4} + \frac{7}{2} = \frac{14 \pm 9\sqrt{2}}{4}$$

8. A particle P is moving along a straight line through the fixed point O
The speed, v m/s, of P at time t seconds is given by

$$v = \frac{t^2}{27} + 2 + \frac{3}{t^2} \quad \text{for} \quad 1 \leqslant t \leqslant 5$$

(a) Complete the table of values for $v = \frac{t^2}{27} + 2 + \frac{3}{t^2}$

Give your values of v to 2 decimal places.

t	1	1.5	2	2.5	3	3.5	4	4.5	5
v	5.04	3.42	2.90	2.71	2.67	2.70	2.78	2.90	3.05

(b) On the grid opposite, plot the points from your completed table and join them to form a smooth curve.

(c) Using your curve, find an estimate, to one decimal place, for the speed of P when $t = 1.75$

$v = 3.1$

The acceleration of P at time t seconds where $1 \leqslant t \leqslant 5$ is a m/s^2

(d) Find an expression for a in terms of t

$v = \frac{t^2}{27} + 2 + \frac{3}{t^2}$ $\quad a = \frac{2t}{27} - 6t^{-3}$

(e) Using your answer to part (d), find the value of t when P has its minimum speed in the time interval $1 \leqslant t \leqslant 5$
Show clear algebraic working.

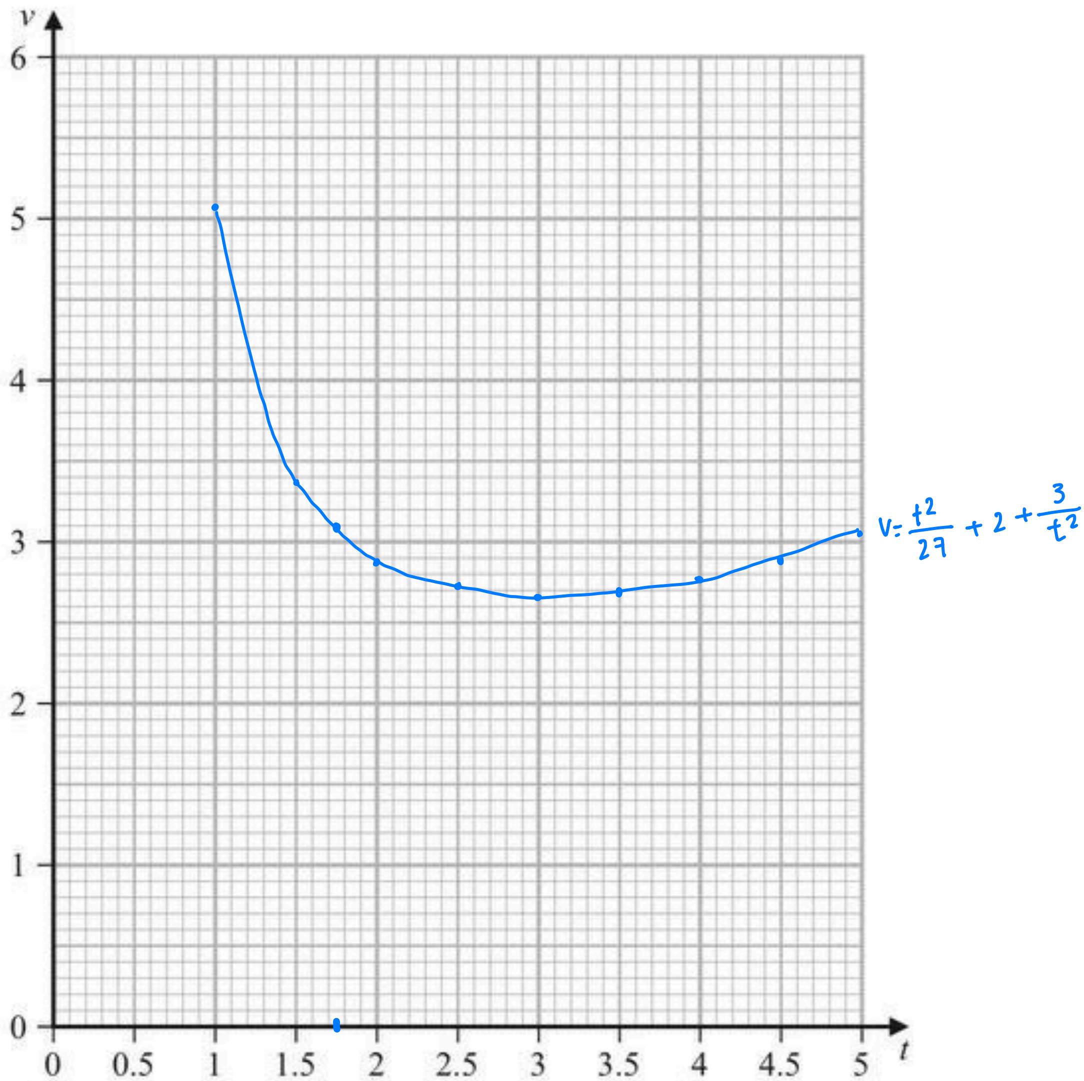
$a = 0$

$\frac{2t}{27} - \frac{6}{t^3} = 0$

$2t^4 = 162$

$t^4 = 81$

$t = 3$


v
6
5
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t
$v = \frac{t^2}{27} + 2 + \frac{3}{t^2}$